Access to Science - Mathematics 1
ADEDEX424
Semester 1 2014-2015 Exam Solutions

1. (i) (a) $\dfrac{3}{7}-\dfrac{4}{5}=\dfrac{(3)(5)+(-4)(7)}{(7)(5)}=\dfrac{-13}{35}=-\dfrac{13}{35}$.

(b) $\dfrac{8}{5}\times\left(-\dfrac{2}{7}\right)=\dfrac{(8)(-2)}{(5)(7)}=\dfrac{-16}{35}=-\dfrac{16}{35}$.

(c) $\dfrac{7}{8}\div\dfrac{8}{7}=\dfrac{7}{8}\times\dfrac{7}{8}=\dfrac{(7)(7)}{(8)(8)}=\dfrac{49}{64}$.

(d) $-2^2=-\left(2^2\right)=-4$.

(e) $\left(\dfrac{3}{4}\right)^{-2}=\dfrac{1}{\left(\dfrac{3}{4}\right)^2}=\dfrac{1}{\dfrac{9}{16}}=\dfrac{16}{9}$.

(f) $\sqrt[3]{8}=2$.

(g) $8^{\frac{5}{3}}=\left(8^{\frac{1}{3}}\right)^5=2^5=32$.

(h) $\left(\dfrac{27}{8}\right)^{-\frac{2}{3}}=\dfrac{1}{\left(\dfrac{27}{8}\right)^{\frac{2}{3}}}=\dfrac{1}{\left(\left(\dfrac{27}{8}\right)^{\frac{1}{3}}\right)^2}=\dfrac{1}{\left(\dfrac{3}{2}\right)^2}=\dfrac{1}{\dfrac{9}{4}}=\dfrac{4}{9}$.

(i) $5+8\times(-7)-4=5+(-56)+(-4)=-55$.

(j) $(3-2)^{-3}\times4+5\times3^2=1^{-3}\times4+5\times3^2=1\times4+5\times3^2=1\times4+5\times9=4+45=49$.

(k) Since $4^2=16$, it follows that $\log_4 16=2$.

(l) Since $5^{-2}=\dfrac{1}{25}$, it follows that $\log_5\dfrac{1}{25}=-2$.

(m) Since $4^{\frac{1}{2}}=2$, it follows that $\log_4 2=\dfrac{1}{2}$. **[13]**

(ii) (a) $x^6\times x^3=x^{6+3}=x^9$.

(b) $x^{-2}\times x^3=x^{-2+3}=x^1=x$.

(c) $x^2\times x^{-\frac{1}{3}}=x^{2+\left(-\frac{1}{3}\right)}=x^{\frac{5}{3}}$.

(d) $x^2\div x^4=x^{2-4}=x^{-2}$.

(e) $x^{\frac{1}{5}}\div x^{-\frac{3}{4}}=x^{\frac{1}{5}-\left(-\frac{3}{4}\right)}=x^{\frac{4+15}{20}}=x^{\frac{19}{20}}$.

(f) $\left(x^{-3}\right)^2=x^{-3\times2}=x^{-6}$.

(g) $\left(x\sqrt[4]{y}\right)^4=\left(x^1\right)^4\left(\sqrt[4]{y}\right)^4=\left(x^1\right)^4\left(y^{\frac{1}{4}}\right)^4=x^{1(4)}y^{\frac{1}{4}(4)}=x^4y$.

(h) $\left(x^{-1}y^{\frac{1}{2}}z^{-\frac{2}{3}}\right)^{-\frac{1}{4}}=\left(\left(x^{-1}y^{\frac{1}{2}}\right)z^{-\frac{2}{3}}\right)^{-\frac{1}{4}}=\left(x^{-1}y^{\frac{1}{2}}\right)^{-\frac{1}{4}}\left(z^{-\frac{2}{3}}\right)^{-\frac{1}{4}}$

$=\left(x^{-1}\right)^{-\frac{1}{4}}\left(y^{\frac{1}{2}}\right)^{-\frac{1}{4}}\left(z^{-\frac{2}{3}}\right)^{-\frac{1}{4}}=x^{-1\left(-\frac{1}{4}\right)}y^{\frac{1}{2}\left(-\frac{1}{4}\right)}z^{-\frac{2}{3}\left(-\frac{1}{4}\right)}=x^{\frac{1}{4}}y^{-\frac{1}{8}}z^{\frac{1}{6}}$. **[8]**

(iii) (a) $17.950 = 18.0$ to one decimal place.

(b) $1234567 = 1235000$ to four significant figures.

(c) $0.000123 = 0.0001$ to one significant figure.

(d) $13214.53 = 1.321453 \times 10^4$ in scientific notation.

(e) $0.00235 = 2.4 \times 10^{-3}$ in scientific notation to two significant figures. **[5]**

(iv) $(x^4 - 2x^3 + 2x - 2) - (x^5 - 2x^4 + 3x^3 - x^2 - 4)$
$= -x^5 + (x^4 + 2x^4) + (-2x^3 - 3x^3) + x^2 + 2x + (-2 + 4)$
$= -x^5 + 3x^4 - 5x^3 + x^2 + 2x + 2.$ **[2]**

(v)

$$\begin{aligned}(x^3 - 2x)(2x^2 + 3) &= (x^3)(2x^2 + 3) + (-2x)(2x^2 + 3)\\ &= (x^3)(2x^2) + (x^3)(3) + (-2x)(2x^2) + (-2x)(3)\\ &= 2x^{3+2} + 3x^3 - 4x^{1+2} - 6x\\ &= 2x^5 + 3x^3 - 4x^3 - 6x\\ &= 2x^5 - x^3 - 6x.\end{aligned}$$

[2]

(vi)

$$\begin{array}{r} 1823\\ 7\,\overline{)\,12766}\\ \underline{7000}\\ 5766\\ \underline{5600}\\ 166\\ \underline{140}\\ 26\\ \underline{21}\\ 5 \end{array}$$

So $\dfrac{12766}{7} = 1823 + \dfrac{5}{7}$.
That is the quotient is 1823 and the remainder is 5. **[3]**

(vii)

$$\begin{array}{r} x - 2\\ x+1\,\overline{)\; x^2 \quad - x + 2}\\ \underline{-\, x^2 \quad - x \phantom{{}+2}}\\ -\,2x + 2\\ \underline{2x + 2}\\ 4 \end{array}$$

This tells us that $\dfrac{x^2 - x + 2}{x + 1} = x - 2 + \dfrac{4}{x + 1}$.
So the quotient is $x - 2$ and the remainder is 4. **[4]**

(viii) $\displaystyle\sum_{i=-2}^{2} i^4 = (-2)^4 + (-1)^4 + 0^4 + 1^4 + 2^4 = 16 + 1 + 0 + 1 + 16 = 34.$ **[2]**

(ix) $\dbinom{9}{3} = \dfrac{9 \times 8 \times 7}{3 \times 2} = 84.$ **[2]**

(x)

$$\begin{aligned}(x-3y)^3 &= x^3 + \binom{3}{1}x^2(-3y) + \binom{3}{2}x(-3y)^2 + (-3y)^3 \\ &= x^3 - 9x^2y + 27xy^2 - 27y^3.\end{aligned}$$

[4]

2. (i) The slope of the line is $m = \dfrac{3-5}{2-(-2)} = \dfrac{-2}{4} = -\dfrac{1}{2}$.

Thus the equation of the line is $y = -\dfrac{1}{2}x + c$, where we still have to find c.

If we substitute $x = -2$ and $y = 5$ into $y = -\dfrac{1}{2}x + c$, we obtain $5 = -\dfrac{1}{2}(-2) + c$.

Thus $c = 5 + \dfrac{1}{2}(-2) = 5 - 1 = 4$.

Hence the equation of the line is $y = -\dfrac{1}{2}x + 4$. **[3]**

(ii) Here our line is parallel to a line that has slope -3, so our line also has slope $m = -3$. Hence the equation of the line is $y = -3x + c$, where we still have to find c. On substituting $x = 2$ and $y = -3$ into $y = -3x + c$, we obtain $-3 = -3(2) + c$, so that $c = -3 + 3(2) = 3$.
Hence the equation of the line is $y = -3x + 3$. **[2]**

(iii) If we subtract two times the second equation from three times the first we obtain

$$\begin{array}{rrcrcr} & 6x & + & -9y & = & -21 \\ - & 6x & + & -4y & = & -6 \\ \hline & & & -5y & = & -15 \end{array}$$

Hence $y = 3$ and on substituting this into the first equation we get
$2x = 3y - 7 = 9 - 7 = 2$, so that $x = 1$.
Thus the solution is $x = 1$ and $y = 3$. **[3]**

(iv) Using $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (2, -1)$, the formula tells us that the length of the line segment is

$$\sqrt{(2-(-1))^2 + (-1-2)^2} = \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}.$$

[1]

(v) If we let $(x_1, y_1) = (2, -3)$ and $(x_2, y_2) = (4, 0)$, then using the formula, we have that the midpoint is

$$\left(\frac{2+4}{2}, \frac{-3+0}{2}\right) = \left(3, -\frac{3}{2}\right).$$

[1]

3. (i)

$$\begin{aligned}2x^2 - x - 10 &= 2\left\{x^2 - \frac{1}{2}x - 5\right\}\\ &= 2\left\{\left(x - \frac{1}{4}\right)^2 - \frac{1}{16} - 5\right\}\\ &= 2\left\{\left(x - \frac{1}{4}\right)^2 - \frac{81}{16}\right\}\\ &= 2\left(x - \frac{1}{4}\right)^2 - \frac{81}{8}.\end{aligned}$$

[3]

(ii) Using the completed square form found in Part (i):

$$\begin{aligned}2x^2 - x - 10 = 0 &\Rightarrow 2\left(x - \frac{1}{4}\right)^2 - \frac{81}{8} = 0\\ &\Rightarrow 2\left(x - \frac{1}{4}\right)^2 = \frac{81}{8}\\ &\Rightarrow \left(x - \frac{1}{4}\right)^2 = \frac{81}{16}\\ &\Rightarrow x - \frac{1}{4} = \pm\frac{9}{4}\\ &\Rightarrow x = -2 \text{ or } x = \frac{5}{2}.\end{aligned}$$

[3]

(iii) In this case $a = 2$, $b = -1$ and $c = -10$.
Hence the solutions of the equation are

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\ &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-10)}}{2(2)}\\ &= \frac{1 \pm \sqrt{1 + 80}}{4}\\ &= \frac{1 \pm \sqrt{81}}{4}\\ &= \frac{1 \pm 9}{4}\\ &= -2 \text{ or } \frac{5}{2},\end{aligned}$$

confirming the answer to Part (ii). **[2]**

(iv) From Part (i) we know that the graph cuts the x-axis when $x = -2$ and when $x = \frac{5}{2}$.
Next, when $x = 0$, $y = -10$, so the graph cuts the y-axis when $y = -10$.

We also know the graph is U-shaped since $a > 0$.
Finally, the turning point is given by

$$\left(-\frac{b}{2a}, -\frac{b^2-4ac}{4a}\right) = \left(-\frac{-1}{2(2)}, -\frac{(-1)^2-4(2)(-10)}{4(2)}\right) = \left(\frac{1}{4}, -\frac{81}{8}\right).$$

We now have all the information we need and I have sketched the graph in Figure 1.

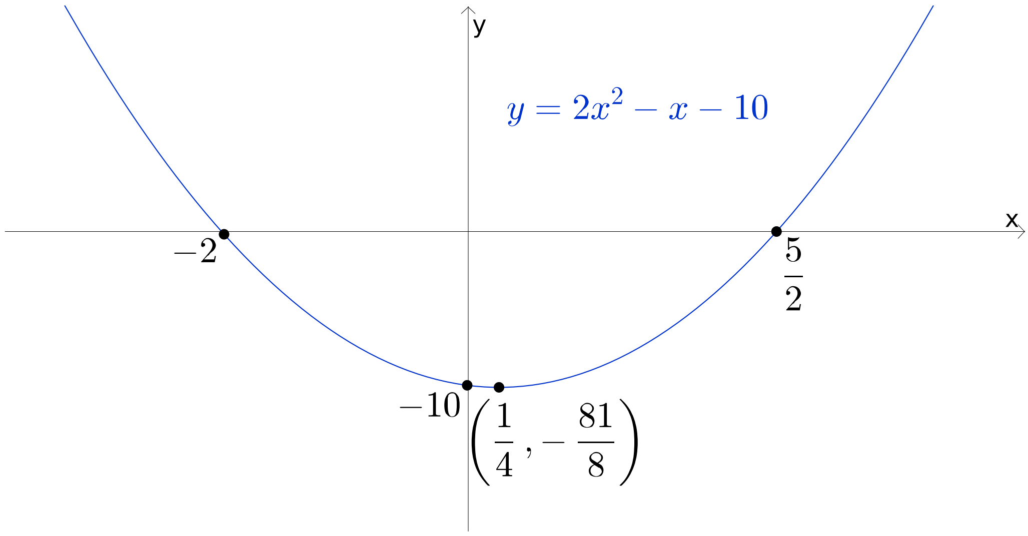

Figure 1: The Graph of the function $y = 2x^2 - x - 10$.

[4]

(v) $2x^2 - x - 10 = 2\,(x-(-2))\left(x-\frac{5}{2}\right) = (x+2)(2x-5)$. **[2]**

4. (i) (a) This is not a function.
For example, $f(0)$ is not defined, since 2 does not lie in the codomain.

(b) This is a function.
Its domain is $\mathbb{R}^-$ and its codomain is $\mathbb{R}^+$.

(c) This is not a function.
For example, $f(0)$ is not defined, since 1 does not lie in the codomain. **[6]**

(ii) (a) Figure 2 shows the graph of the function

$$\begin{aligned} f\colon \{-3,-2,0,2,3\} &\to \{1,2,3\} \\ -3 &\mapsto 3 \\ -2 &\mapsto 2 \\ 0 &\mapsto 1 \\ 2 &\mapsto 2 \\ 3 &\mapsto 3 \end{aligned}$$

(b) Figure 3 shows the graph of the function

$$\begin{aligned} f\colon \{x \in \mathbb{R}\colon -3 \leqslant x \leqslant 2\} &\to \{x \in \mathbb{R}\colon -10 \leqslant x \leqslant 10\} \\ x &\mapsto 2x-3 \end{aligned}$$

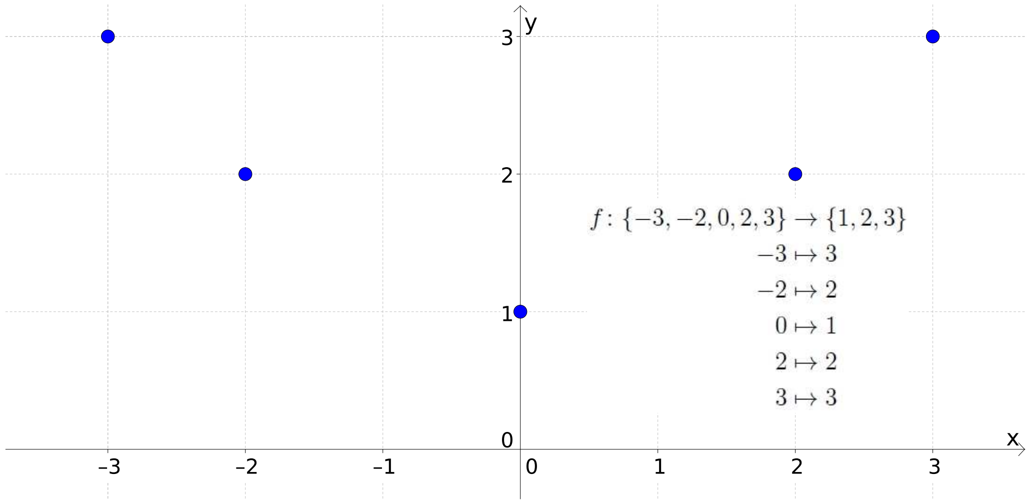

Figure 2: The graph of the function defined in Question 4(ii)(a).

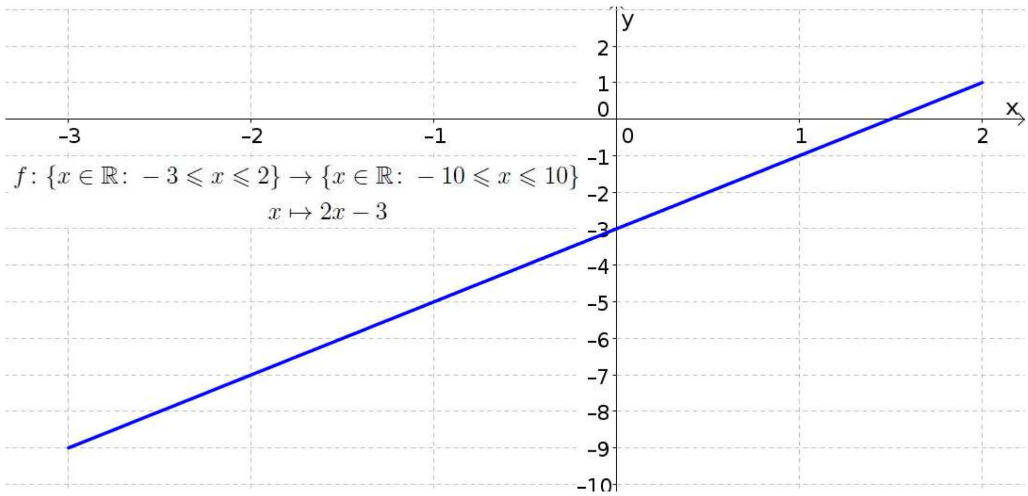

Figure 3: The graph of the function defined in Question 4(ii)(b).

[4]

(iii) The functions h and k both cross the x-axis so these must be the log functions. The function k increases as x increases, so k is the function in (d). Then the function h must be the function in (e). Next g lies below the x-axis, so it must be one of (a) or (c). Now $y = 7^x$ increases as x increases, so $y = -7^x$ decreases as x increases. Hence g can't be (a) and so must be (c). Finally l must be (b) or (f). However $y = \left(\frac{5}{7}\right)^x$ decreases as x increases, so it can't be l. Thus l must be (f).
Summarizing: g is (c), h is (e), k is (d) and l is (f). **[4]**

(iv) (a) This function is injective.
It is not surjective since there is no x with $f(x) = 5$.
It is not bijective since it is not surjective.

(b) This function is injective, surjective and hence bijective.

(c) This function is injective.
It is not surjective since there is no x with $f(x) = 0$.
It is not bijective since it is not surjective. **[5]**

(v) The inverse function of the function in Question 4(iv)(b) is:

$$\begin{aligned} f^{-1} \colon \{1, 2, 3, 4\} &\to \{A, B, C, D\} \\ 1 &\mapsto C \\ 2 &\mapsto D \\ 3 &\mapsto B \\ 4 &\mapsto A \end{aligned}$$

[2]

5. (i) $345° = 345 \times \dfrac{\pi}{180} = \dfrac{23\pi}{12}$ Radians. **[1]**

(ii) $\dfrac{5\pi}{12}$ Radians $= \left(\dfrac{5\pi}{12} \times \dfrac{180}{\pi}\right)^{\circ} = 75°$. **[1]**

(iii) In this case we want to find $\sin(\theta)$ when $\theta = \dfrac{5\pi}{4}$.

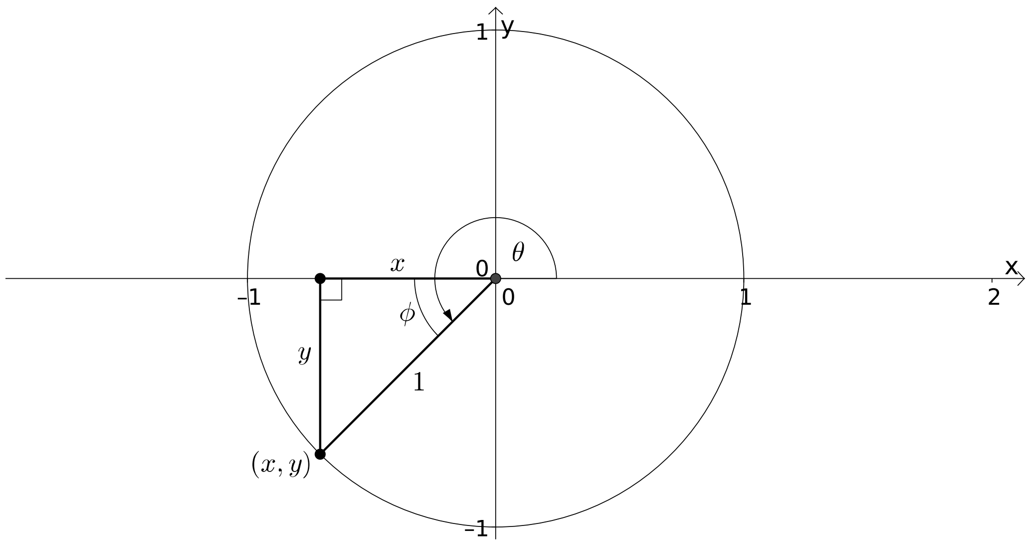

Figure 4: Calculation of $\sin\left(\dfrac{5\pi}{4}\right)$.

Looking at Figure 4, we see that we need to find y, since this is by definition $\sin\left(\dfrac{5\pi}{4}\right)$. Now, also from Figure 4, $\phi = \dfrac{5\pi}{4} - \pi = \dfrac{\pi}{4}$ (where we are just treating ϕ as an angle rather than a directed angle). Hence, using the table of common values, $\sin(\phi) = \dfrac{1}{\sqrt{2}}$. But also by definition $\sin(\phi) = |y|$ (since the hypotenuse has length 1). Now, since y is negative, $y = -|y|$ and so $\sin\left(\dfrac{5\pi}{4}\right) = -\dfrac{1}{\sqrt{2}}$. **[3]**

(iv) (a) Here we will first use $\sin(\theta) = \cos\left(\dfrac{\pi}{2} - \theta\right)$.
We have $\sin\left(\dfrac{2\pi}{3}\right) = \cos\left(\dfrac{\pi}{2} - \dfrac{2\pi}{3}\right) = \cos\left(-\dfrac{\pi}{6}\right)$.
Next we will use $\cos(-\theta) = \cos(\theta)$ and our table of common values to obtain $\cos\left(-\dfrac{\pi}{6}\right) = \cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$. Hence $\sin\left(\dfrac{2\pi}{3}\right) = \dfrac{\sqrt{3}}{2}$.

(b) We will first use the fact that the cosine function repeats every 2π.
Thus $\cos\left(\dfrac{5\pi}{3}\right) = \cos\left(\dfrac{5\pi}{3} - 2\pi\right) = \cos\left(-\dfrac{\pi}{3}\right)$.
We can now use our table of common values and $\cos(-\theta) = \cos(\theta)$ to obtain $\cos\left(-\dfrac{\pi}{3}\right) = \cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$. Hence $\cos\left(\dfrac{5\pi}{3}\right) = \dfrac{1}{2}$.

(c) Here we will use $\tan(A - B) = \dfrac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}$ with $A = \dfrac{\pi}{4}$ and $B = \dfrac{\pi}{3}$.
Hence

$$\tan\left(-\frac{\pi}{12}\right) = \frac{\tan\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{3}\right)}{1 + \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{3}\right)} = \frac{1 - \sqrt{3}}{1 + (1)(\sqrt{3})} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}.$$

[6]

(v) Using the cosine in the form $a^2 = b^2 + c^2 - 2bc\cos(A)$ we obtain
$a^2 = 8^2 + 10^2 - 2(8)(10)\cos(71^\circ) \simeq 111.91$.
Hence $a \simeq \sqrt{111.91} \simeq 10.58$. **[3]**

6. (i)

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{(x+h)^2 + 1 - (x^2+1)}{h} \\
&= \lim_{h\to 0} \frac{(x^2 + 2xh + h^2) + 1 - (x^2+1)}{h} \\
&= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 + 1 - x^2 - 1}{h} \\
&= \lim_{h\to 0} \frac{2xh + h^2}{h} \\
&= \lim_{h\to 0} 2x + h \\
&= 2x.
\end{aligned}$$

[3]

(ii) (a) $f'(x) = 0.$

(b) $f'(x) = 0.$

(c) $f'(x) = 4x^3.$

(d) $f'(x) = \dfrac{1}{x}.$

(e) $f'(x) = -3\cos(-3x).$

(f) $f'(x) = 0 - 4\left(-2x^{-2-1}\right) + 2\left(\dfrac{4}{5}x^{\frac{4}{5}-1}\right) = 8x^{-3} + \dfrac{8}{5}x^{-\frac{1}{5}}.$

(g) $f'(x) = 3\left(-2\sin(2x)\right) - 4\left(-\cos(-x)\right) = -6\sin(2x) + 4\cos(-x).$

(h) $f'(x) = 0 + 4\left(-5e^{-5x}\right) + 4\left(\dfrac{1}{x}\right) = -20e^{-5x} + \dfrac{4}{x}.$ **[11]**

7. (i) $\displaystyle\int 6\,dx = 6x + c.$ **[1]**

(ii) $\displaystyle\int_1^2 x^7\,dx = \left[\frac{1}{8}x^8\right]_1^2 = \frac{1}{8}(2^8) - \frac{1}{8}(1^8) = \frac{255}{8}.$ **[2]**

(iii) $\displaystyle\int \cos(4x)\,dx = \frac{1}{4}\sin(4x) + c.$ **[1]**

(iv)

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} \sin(-3x)\,dx &= \left[\frac{1}{-3}(-\cos(-3x)\right]_0^{\frac{\pi}{2}} \\
&= \left[\frac{1}{3}(\cos(-3x)\right]_0^{\frac{\pi}{2}} \\
&= \frac{1}{3}\cos\left(-\frac{3\pi}{2}\right) - \frac{1}{3}\cos(0) \\
&= 0 - \frac{1}{3} \\
&= -\frac{1}{3}.
\end{aligned}$$

[3]

(v)

$$\begin{aligned}
\int 2 - 3x^2 + 2x^{\frac{3}{4}}\,dx &= 2x - 3\left(\frac{1}{3}\right)x^3 + 2\left(\frac{1}{7/4}\right)x^{\frac{7}{4}} + c \\
&= 2x - x^3 + \frac{8}{7}x^{\frac{7}{4}} + c.
\end{aligned}$$

[2]

(vi)

$$\begin{aligned}\int_1^2 5-3e^{-5x}\,dx &= \left[5x-3\frac{1}{-5}e^{-5x}\right]_1^2\\ &= \left[5x+\frac{3}{5}e^{-5x}\right]_1^2\\ &= 10+\frac{3}{5}e^{-10}-\left(5+\frac{3}{5}e^{-5}\right)\\ &= 5+\frac{3}{5}\left(e^{-10}-e^{-5}\right).\end{aligned}$$

[3]

8. (i) (a) The mean is

$$\begin{aligned}\overline{x} &= \frac{1}{10}(3+2+(-8)+(-4)+8+3+2+5+(-7)+(-5))\\ &= \frac{-1}{10}\\ &= -\frac{1}{10}.\end{aligned}$$

(b) The list in ascending order is $-8,-7,-5,-4,2,2,3,3,5,8$.
Since there are ten numbers (an even number), the median is
$m=\dfrac{x_{\frac{10}{2}}+x_{\frac{10}{2}+1}}{2}=\dfrac{x_5+x_6}{2}=\dfrac{2+2}{2}=2.$

(c) There are 2 twos, 2 threes and one of each of the other numbers, so there are two modes, 2 and 3.

(d) The variance is (where I have used the original ordering)

$$\begin{aligned}\mathrm{Var}(x) =&\frac{\sum_{i=1}^{10}(x_i-\overline{x})^2}{10}\\ =&\frac{\left(3-\left(-\frac{1}{10}\right)\right)^2+\left(2-\left(-\frac{1}{10}\right)\right)^2+\left(-8-\left(-\frac{1}{10}\right)\right)^2}{10}\\ &+\frac{\left(-4-\left(-\frac{1}{10}\right)\right)^2+\left(8-\left(-\frac{1}{10}\right)\right)^2+\left(3-\left(-\frac{1}{10}\right)\right)^2}{10}\\ &+\frac{\left(2-\left(-\frac{1}{10}\right)\right)^2+\left(5-\left(-\frac{1}{10}\right)\right)^2+\left(-7-\left(-\frac{1}{10}\right)\right)^2}{10}\\ &+\frac{\left(-5-\left(-\frac{1}{10}\right)\right)^2}{10}\\ =&\frac{2689/10}{10}\\ =&\frac{2689}{100}\\ =&26.89.\end{aligned}$$

(e) The standard deviation is $\sigma = \sqrt{\text{Var}(x)} = \sqrt{\frac{2689}{100}} \simeq 5.186$.

(f) Since there are ten numbers (an even number) we just split the numbers into a lower half $-8, -7, -5, -4, 2$ and an upper half $2, 3, 3, 5, 8$.
There are five numbers in each of these new groups (an odd number), so in each case the median is $x_{\frac{5+1}{2}} = x_3$.
Hence the lower quartile is $Q_1 = -5$ and the upper quartile is $Q_3 = 3$.
Thus the interquartile range is $Q_3 - Q_1 = 3 - (-5) = 8$. **[8]**

(ii) There are seven points, so $n = 7$ and

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{7} x_i = -4 + (-2) + 0 + 1 + 2 + 5 + 6 = 8$$
$$\sum_{i=1}^{n} y_i = \sum_{i=1}^{7} y_i = -1 + 0 + 0 + 1 + 2 + 3 + 4 = 9$$

$$\begin{aligned}\sum_{i=1}^{n} x_i y_i = &\sum_{i=1}^{7} x_i y_i \\ =&(-4)(-1) + (-2)(0) + (0)(0) + (1)(1) + (2)(2) + (5)(3) + (6)(4) \\ =&4 + 0 + 0 + 1 + 4 + 15 + 24 \\ =&48.\end{aligned}$$

$$\begin{aligned}\sum_{i=1}^{n} x_i^2 = &\sum_{i=1}^{7} x_i^2 \\ =&(-4)^2 + (-2)^2 + 0^2 + 1^2 + 2^2 + 5^2 + 6^2 \\ =&16 + 4 + 0 + 1 + 4 + 25 + 36 \\ =&86.\end{aligned}$$

Hence

$$\begin{aligned}m &= \frac{n\left(\sum_{i=1}^{n} x_i y_i\right) - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n\left(\sum_{i=1}^{n} x_i^2\right) - \left(\sum_{i=1}^{n} x_i\right)^2} \\ &= \frac{7(48) - (8)(9)}{7(86) - 8^2} \\ &= \frac{264}{538} \\ &= \frac{132}{269} \\ &\simeq 0.491,\end{aligned}$$

and

$$c = \overline{y} - m\overline{x} = \frac{\sum_{i=1}^{7} y_i}{7} - m\frac{\sum_{i=1}^{7} x_i}{7} = \frac{9}{7} - \frac{132}{269} \times \frac{8}{7} = \frac{195}{269} \simeq 0.725.$$

Thus the line of best fit is $y = \frac{132}{269}x + \frac{195}{269}$.

The points and the graph are shown in Figure 5.

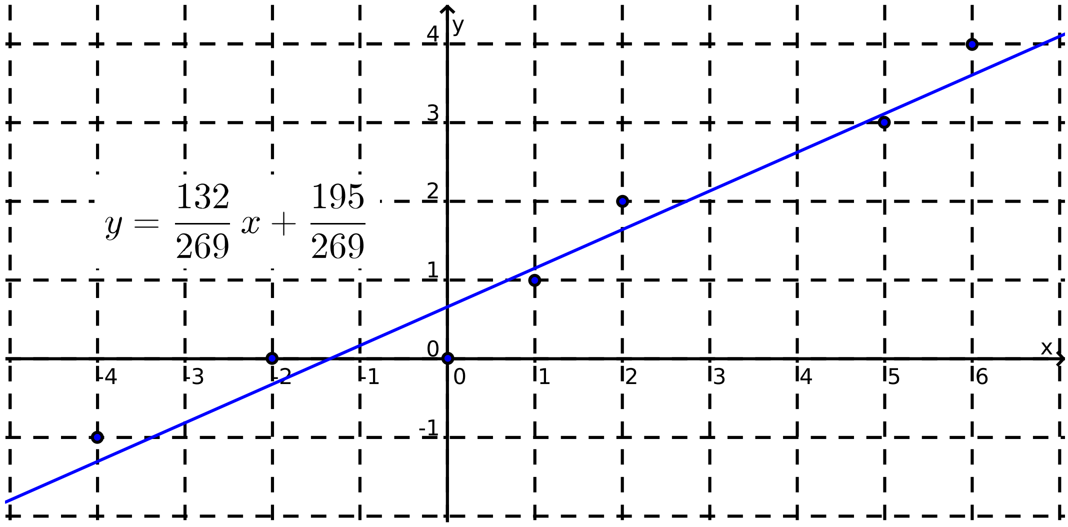

Figure 5: The Line of Best Fit and Points For Question 8(ii).

[12]